PHY412: Stellar Structure and evolution
Example exam questions

1. Describe the evolution of a typical star of 1 solar mass. You should describe the phases from the beginning of the main sequence to the entry of the planetary nebula phase.

   Explain what is meant by the term polytrope.

   Suppose that a spherical star has the equation of state

   $$P = K\rho^2$$

   Show that the Lane-Emden equation has a root

   $$\theta = \frac{\sin\xi}{\xi}$$

   Discuss the physical meaning of this solution and the application of the Lane-Emden equation.

   Note the Lane - Emden equation is :

   $$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n \qquad \text{where } r = \alpha\xi$$

2. Derive the condition for the onset of convection in a stellar interior

   $$\frac{P}{T}\frac{dT}{dP} > \frac{\gamma-1}{\gamma}$$

   Where P is the pressure, T the temperature and γ the ratio of specific heats

   Explain briefly why, in a convective core, it is usually adequate to replace the above inequality with the equation

   $$\frac{P}{T}\frac{dT}{dP} = \frac{\gamma-1}{\gamma}$$

   Discuss the physical conditions in which convection may occur in stellar interiors. You should mention the types of stars that may have convective regions.

3. A massive star joins the main-sequence when it has a mass of 15 solar masses. Describe the evolution of this star from this point until its death. Sketch the movement of the star in a labelled Hertzprung-Russell diagram.

Such a star will die in a supernova explosion. Give a brief over-view of this phenomenon, and the type of supernova that would be observed.

4. Derive the equation of hydrostatic equilibrium and the equation of mass conservation in a spherically symmetric, isolated, static star. In both cases define the pressure and mass in terms of the variable r (the radius).

   Use these two equations to derive the Virial Theorem.

   Assume that it is composed of an ideal gas of uniform mean particle mass, that there is negligible radiation pressure, and that the gas pressure vanishes at its surface. Prove the following results:

$$P_c > \frac{GM_S^2}{8\pi r_s^4}$$

$$T_{av} > \frac{GM_S m}{6kr_S}$$

   Where P and T are the pressure and temperature at radius r. M is the mass contained within radius r and the suffixes c and s refer to central and surface values. T_{av} is the mean temperature of the star defined by:

$$M_S T_{av} = \int_o^{M_S} T dM$$

   Use the above relations to determine the minimum central pressure and minimum mean temperature of the Sun.

5. Describe the physical mechanism that creates a thermonuclear supernova. Sketch the lightcurve of a typical type Ia supernova and discuss the energy source that provides the luminosity when the supernova is more than a month old.

6. The equations of stellar structure (where all symbols have their usual meaning) are :

$$\frac{dr}{dM} = \frac{1}{4\pi r^2 \rho} \qquad \frac{dL}{dM} = \varepsilon \qquad P = \frac{\Re \rho T}{\mu}$$

$$\frac{dP}{dM} = -\frac{GM}{4\pi r^4} \qquad \frac{dT}{dM} = \frac{3\kappa_R L}{64\pi^2 r^4 \sigma T^3} \qquad \kappa = \kappa_0 \rho^\alpha T^\beta$$

$$\varepsilon = \varepsilon_0 \rho T^\eta$$

Derive equivalent expressions for a homologous series of models, and the set of 5 algebraic equations in terms of α, β and η.

Assuming α=1, β=-3.5 and η=4, derive a relationship between stellar luminosity and stellar mass for this homologous series of models. Further derive a relationship between stellar luminosity and temperature. Comment on the how you would test the validity of the latter relation.

7. Discuss the factors that govern hydrogen fusion in stellar interiors.
Outline the series of reactions that constitute the "proton-proton chain" and the "carbon-nitrogen cycle".
What factors determine which of these two groups of reactions makes the predominant contribution to energy generation in a star?
Distinguish the influence of the two groups of reactions on the behaviour of the cores of main sequence

8. The equation of radiation transport is given by:

$$\frac{dT}{dr} = \frac{3\rho\kappa_R}{64\pi r^2 \sigma T^3} L(r)$$

Assuming a spherically symmetric star, express this equation in terms of the mass variable *m* instead of *r*

Given the relation for radiative pressure below, derive the expression for the Eddington limit for stellar luminosity and discuss its implications.

$$P_{rad} = \frac{1}{3} a T^4$$

Assuming a typical mass-luminosity relation for massive stars, estimate the upper mass and luminosity limit for the main-sequence. Sketch a typical upper mass main-sequence on a labeled HR diagram and mark on the Eddington limit.