THE ASTROPHYSICAL JOURNAL, 687: L45–L48, 2008 November 1

Ⓔ

# REFINED MAGNETOSEISMOLOGICAL TECHNIQUE FOR THE SOLAR CORONA

G. VERTH AND R. ERDÉLYI
Solar Physics and Space Plasma Research Centre (SP$^2$RC), University of Sheffield, Hicks Building, Hounsfield Road, Sheffield S3 7RH, England, UK;
G.Verth@sheffield.ac.uk, robertus@sheffield.ac.uk
AND
D. B. JESS
Astrophysics Research Centre, School of Mathematics and Physics, Queen's University, Belfast BT7 1NN, Northern Ireland, UK; and NASA Goddard Space Flight Center, Solar Physics Laboratory, Code 671, Greenbelt, MD 20771; djess01@qub.ac.uk

*Received 2008 July 7; accepted 2008 September 16; published 2008 October 3*

## ABSTRACT

We present a step-by-step guide of a refined magnetoseismological technique for plasma diagnostics in the Sun's corona. Recently developed MHD wave theory which models a coronal loop as an expanding magnetic flux tube with an arbitrary longitudinal plasma density profile is applied to *TRACE* observations of fast kink oscillations. The theory predicts that using the observed ratio of the first overtone and fundamental mode to predict the plasma density scale height and not taking account of loop expansion will lead to an overestimation of scale height. For the first time, the size of this correction is quantified directly from observation and for the particular case study presented here, it is found that the overestimation is approximately by a factor of 2.

*Subject headings:* MHD — Sun: corona — Sun: magnetic fields — Sun: oscillations

*Online material:* color figure

## 1. INTRODUCTION

The pioneering suggestions of, e.g., Uchida (1970) and Roberts et al. (1984) to perform magnetoseismology of the solar corona as an alternative indirect plasma diagnostic method have now come to fruition with the advent of high temporal/spatial resolution space-borne instruments. A prime example of this is the high-resolution EUV imager on board the *Transition Region And Coronal Explorer* (*TRACE*), launched in 1998, which allowed us for the first time to directly observe postflare transversal coronal loop oscillations. These were conclusively identified as the standing fast kink body mode from magnetohydrodynamic (MHD) wave theory (see, e.g., Aschwanden et al. 1999; Nakariakov et al. 1999). The theory of MHD waves in magnetic cylinders was first developed by, e.g., Edwin & Roberts (1983) and assumed a relatively simple equilibrium of different plasma density and magnetic field strength inside and outside the wave guide, all of which were taken to be constants. Initial magnetoseismological estimates of coronal magnetic field strength were made applying this theory to *TRACE* observations by Nakariakov & Ofman (2001).

Although still assuming a constant magnetic field, the theory of Edwin & Roberts has since been further advanced by allowing arbitrary longitudinal plasma density profiles inside and outside the cylinder (see, e.g., Díaz et al. 2002; Andries et al. 2005b; Dymova & Ruderman 2005; Erdélyi & Verth 2007; Verth et al. 2007). This lead to attempts of estimating the density scale height in the corona using the observed frequency ratio of the first overtone and the fundamental mode ($\omega_2/\omega_1$), which depends on the degree of density stratification along a loop (see, e.g., Andries et al. 2005a; Van Doorsselaere et al. 2007). Specifically, when the plasma in a coronal loop is gravitationally stratified, $\omega_2/\omega_1 < 2$ and the greater the degree of stratification, the smaller this ratio becomes. More recently, the effect of magnetic flux tube expansion on the fast kink mode has been addressed by Verth & Erdélyi (2008, hereafter VE08) and Ruderman et al. (2008). On average, plasma density and magnetic strength should decrease with height above the photosphere. Due to the reciprocal nature of magnetic field strength and plasma density in the kink speed, this leads to the effect of magnetic stratification increasing the value of $\omega_2/\omega_1$. In using observed values of $\omega_2/\omega_1$ to estimate the degree of plasma density stratification in coronal loops, it has been shown by VE08 that failure to take account of the flux tube expansion causes the density scale height to be overestimated. Furthermore, there have been observations of $\omega_2/\omega_1 > 2$ (see, e.g., De Moortel & Brady 2007; O'Shea et al. 2007) which could be interpreted as loops having a negative scale height, i.e., being more dense at their apex. However, it was proposed by VE08 that the effect of an expanding flux tube offers a physically more reasonable and simpler explanation for these observations. Implementing this recent MHD wave theory, in this Letter we shall demonstrate a refined magnetoseismological technique that corrects for the effect of loop expansion, thereby allowing us to obtain a more accurate estimate of plasma density scale height in the corona.

## 2. MHD WAVE THEORY AND OUTLINE OF TECHNIQUE

Using cylindrical coordinates, VE08 modeled an expanding magnetic flux tube with arbitrary internal and external longitudinal density profiles. In conjunction with normal mode analysis, the relatively small loop expansions observed in EUV by *TRACE* (see Watko & Klimchuk 2000) and the fact that fast kink modes have wavelengths which are much greater than loop width permitted them to asymptotically simplify the governing MHD wave equations. The second-order ODE governing the velocity of the fast kink mode they derived (see their eq. [67]) was recently verified by Ruderman et al. (2008) using magnetic flux coordinates, showing that this equation can be expressed in the relatively simple form

$$\frac{d^2}{ds^2}[B^{1/2}(s)v(s)] + \left[\frac{\omega}{c_k(s)}\right]^2 B^{1/2}(s)v(s) = 0, \qquad (1)$$

where $s$ is the coordinate along the loop, $v$ is the transverse velocity, $B$ is the magnetic field strength, $\omega$ is the angular

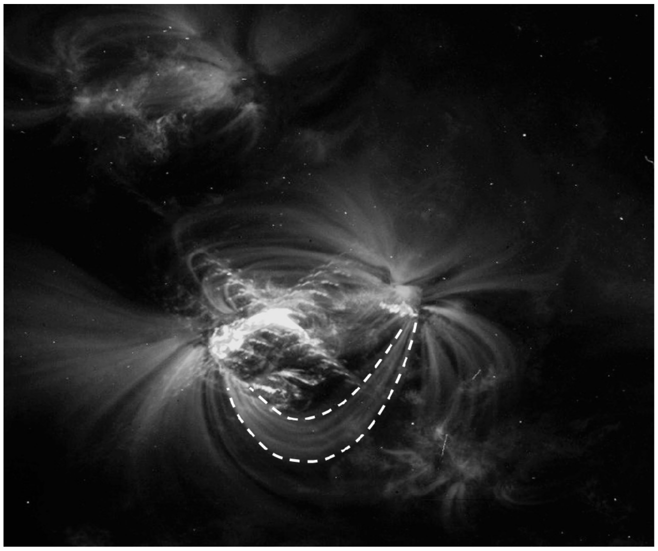

FIG. 1.—*TRACE* image in 171 Å at 12:57 UT, 2 minutes after the onset of the flare. Note that the loops in the marked southern half of the active region are bright in intensity and clearly visible along most of their lengths.

frequency, and $c_k$ is the kink speed defined by $c_k^2(s) = 2B^2(s)/\{\mu[\rho_i(s) + \rho_e(s)]\}$, where $\mu$ is magnetic permeability and $\rho_i$ ($\rho_e$) is the internal (external) density.

The specific aim of the magnetoseismological technique presented in this Letter is to deduce the coronal plasma density scale height $H$ from the observed $\omega_2/\omega_1$ ratio, correcting for loop shape, inclination, and expansion. The effective density scale height of a loop can be strongly influenced by loop shape and inclination (see, e.g., Dymova & Ruderman 2006; Aschwanden et al. 2008a); therefore to make an accurate estimate of density scale height one must correct for geometric effects. The recently launched *STEREO* (*Solar TErrestrial RElations Observatory*) should be a great help in this regard. Already some progress has been made by Aschwanden et al. (2008b) using *STEREO* data to employ triangulation techniques in reconstructing 3D loop geometry and quantifying possible effects such as loop non-circularity and non-coplanarity. Since in the following analysis we are restricted to *TRACE* data predating the *STEREO* launch, instead we model loop shape and inclination using the circular fit model of Aschwanden et al. (2002). It is important to stress at this point that the basic principles of our proposed magnetoseismological technique will remain the same, regardless of the which method is employed to make an accurate reconstruction of a loop's 3D geometry. In our chosen circular fit model, $\theta$ is the loop plane inclination from vertical, $r$ is the radius, and $h_0$ denotes the distance from the geometric center to the photosphere in the loop plane.

For the plasma density equilibrium we assume that $\rho_i \propto \rho_e$. Let the loop length be $2L$, such that $s \in [-L, L]$, then for a gravitationally, i.e., vertically stratified plasma,

$$\rho_i(s) \propto \exp\left\{-\frac{r(1+\eta)\cos(\theta)\,[\cos(\psi_0 s/L) - \eta]}{H(1-\eta)}\right\}, \quad (2)$$

where $\eta = h_0/r$ and $\psi_0 = \arccos(\eta)$.

The loop expansion factor defined by, e.g., Klimchuk (2000) is given by $\Gamma = r_a/r_f$, where $r_a$ ($r_f$) is the apex (footpoint) radius. Regarding the choice of magnetic field equilibrium in our model, we choose a potential magnetic field of the form given by VE08,

$$B(s) \propto 1 + \frac{(1-\Gamma^2)}{\Gamma^2}\,\frac{\cosh(\kappa s/L) - \cosh(\kappa)}{1 - \cosh(\kappa)}, \quad (3)$$

where $\kappa$ is a parameter that can be used to adjust the magnetic field profile between the footpoint and apex, e.g., to fix a particular loop width expansion at the midpoints, $\pm L/2$. We have not included the possibility of twist in our magnetic equilibrium as Ruderman (2007) has shown that for an approximately force-free cylinder with a uniform weakly twisted magnetic field, there is no effect on the frequency of the fast kink mode.

Significantly, it has been shown by, e.g., Andries et al. (2005a) and VE08 that the ratio $\omega_2/\omega_1$, which we shall be using for our magnetoseismological inversion, is independent of any constants of proportionality for equations (2) and (3). The basic steps for our proposed refined magnetoseismological technique to deduce the unknown value of $H$ are as follows:

*Step 1.*— Measure the frequency ratio $\omega_2/\omega_1$ of the fast kink mode.

*Step 2.*— Estimate geometric parameters determining loop shape, inclination, and expansion in the chosen equilibrium model (all the parameters except $H$ in eqs. [2] and [3] for the specific model chosen here).

*Step 3.*— Use the information obtained from steps 1 and 2 in equation (1) to perform the magnetoseismological inversion and obtain a value for $H$.

## 3. CASE STUDY

The observational example chosen to demonstrate the powerful capability of the theory described in § 2 is an oscillating loop in NOAA active region 8270 observed by *TRACE* after an M4.6 flare on Bastille Day, 1998 July 14, 12:55 UT.[1] Fast kink oscillations of this active region were first reported and analyzed by Aschwanden et al. (1999) and Nakariakov et al. (1999). Following these initial works, seven different kink oscillation events of AR 8270 were investigated in detail by Aschwanden et al. (2002). The particular loop we focus on and analyze here is given as case 1e. Immediately before and after the flare onset, the loop is bright in intensity and clearly visible along most of its length (see Fig. 1). However, during the observed oscillatory phase (approximately 15 minutes after the flare onset) it has undergone a dramatic dimming and is only visible at a small apex region. Importantly, it was shown by Aschwanden et al. (1999), using simultaneous *Yohkoh* SXT data, that this observed dimming is not due to an increase in plasma temperature.

In relation to the magnetic field structure of this active region, it can be seen from Figure 1, that it is predominantly dipolar in nature. There have been several potential extrapolations of this region using Kitt Peak line-of-sight magnetogram data by, e.g., Aulanier et al. (2000) and Ofman (2007) which successfully capture the large-scale features of the observed field lines traced out by the frozen-in plasma seen in the *TRACE* images. In the rest of this section we implement the three steps of our proposed magnetoseismological technique outlined in § 2.

*Step 1.*—The fundamental fast kink mode period of this loop

---

[1] High-resolution movies of the event are available at http://trace.lmsal.com/POD/looposcillations/paperI/SchrijverAschwandenTitle.html in both the 171 Å and 195 Å passbands (given as event No. 1).

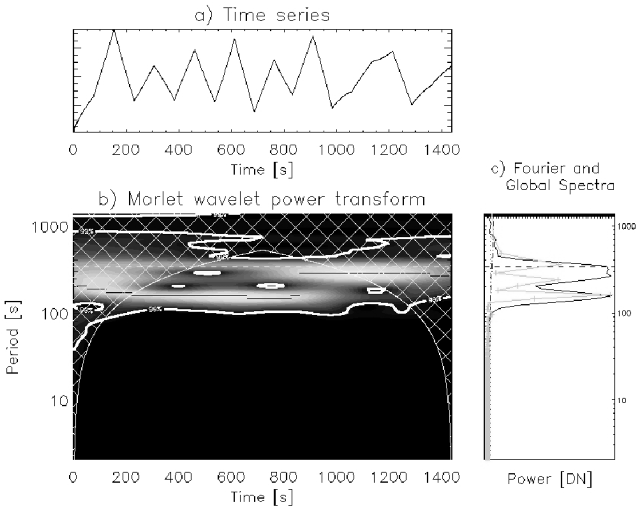

FIG. 2.—Plot at the top left shows the data time series for oscillation perpendicular to the loop. The bottom left plot shows the Morelet wavelet power transform, with the thin and thick white lines indicating the cone of influence and the 99% confidence level contour, respectively. The fundamental and first overtone of the fast kink mode periods can be clearly seen with $P_1 = 242.51$ s and $P_2 = 157.25$ s. The plot on the bottom right shows Fourier (*green line*) and global (*black line*) power spectra. [*See the electronic edition of the Journal for a color version of this figure.*]

was estimated by Aschwanden et al. (2002) to be $P_1 = 242$ s. Investigation of the same periodic intensity perturbations perpendicular to the oscillating loop, through use of feature tracking and wavelet analysis (see Jess et al. 2008 for details of these routines), reveals $P_1 = 242.51$ s, in good agreement with Aschwanden et al. (2002). Furthermore, and most importantly for diagnostic purposes, the first overtone is also detected with $P_2 = 157.25$ s (see Fig. 2), giving $\omega_2/\omega_1 = 1.54$.

*Step 2.*—Reconstructing the shape and inclination of the loop, Aschwanden et al. (2002) estimated that $r = 51$ Mm, $h_0 = 7$ Mm, $L = 87$ Mm, and $\theta = -45°$. Since $h_0/r = 0.14$, to a reasonable approximation, the loop is nearly semicircular in shape and $\theta = -45°$ shows that it has a relatively large inclination angle. To measure the loop expansion we analyzed the *TRACE* image taken at 12:57 UT, approximately 2 minutes after the flare onset when the loop is bright in intensity and is seen clearly from apex to footpoint (shown in Fig. 1). A Gaussian fit to the loop cross-section intensity gives $\Gamma = 2.05$ (see Fig. 3). For an ideal dipole source, an approximately semicircular loop will have $\Gamma \approx 2$. This, along with the visual evidence from the previously mentioned potential field extrapolations, is an indicator that the large-scale field is probably close to potential. Hence, the potential equilibrium magnetic field of the theory given by equation (3) can be considered to be a reasonably valid assumption for our chosen loop. Measuring the midpoint expansion to be $0.71\Gamma$, this determines that $\kappa = 3.47$ in equation (3).

*Step 3.*—Solving the eigenvalue problem of equation (1) governing the kink oscillations of an expanding loop, using the observed $\omega_2/\omega_1$ and geometrical parameters in equations (2) and (3), results in $H = 7.65$ Mm. It can be shown that the coronal density scale height $H$ is related to temperature $T$ by the following relationship, $H = 47\,(T/\mathrm{MK})$ Mm (see, e.g., Aschwanden 2004), hence our estimated $H$ gives a loop temperature diagnostic of $T = 0.16$ MK.

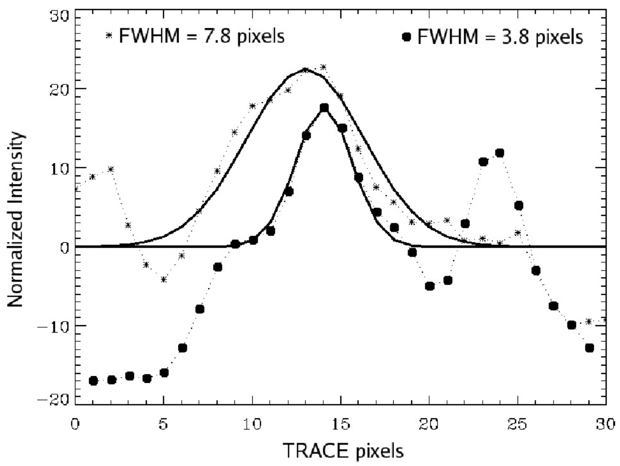

FIG. 3.—On the same plot, we show the intensity profiles for the perpendicular loop cross sections at the apex (*asterisks*) and footpoint (*filled black circles*). The horizontal axes are given in *TRACE* pixels (1 *TRACE* pixel is equivalent to 357 km). Gaussian fits, shown by the solid line, give the full width at half-maximum (FWHM) as 7.8 pixels for the apex and 3.8 pixels for the footpoint.

## 4. DISCUSSION OF RESULT

The *TRACE* 171 Å passband detects EUV emission with a peak response at approximately 1 MK and a smaller secondary peak between about 0.2–0.3 MK (Aschwanden 2004). The diagnosed temperature of $T = 0.16$ MK is therefore near the lower limit of that which can be detected by the 171 Å passband and explains why the loop is so dim during the observed oscillatory phase. To our knowledge, this is the first time that such a relatively cool loop has been identified by a magnetoseismological technique using *TRACE* data. Previously, such loops of $T \approx 0.2$ MK have been diagnosed with *TRACE* using a 171 Å and 195 Å passband filter ratio method (Testa et al. 2002).

Schrijver (2001) in his study of catastrophic loop cooling, using multipassband *TRACE* data, observed loops that cooled from 1.5 MK to less than 0.1 MK in about 15 minutes. The fact that the loop analyzed here is bright in intensity and clearly visible immediately before and after the flare means that it is likely to be near the 171 Å peak response temperature of 1 MK. Then the temperature drop of about 0.8 MK and cooling time of 15 minutes (from the flare onset to the observed oscillatory phase) are both of the order of that observed by Schrijver (2001). There is also further direct observational evidence to support our cool loop diagnosis since from 13:21 UT plasma is seen sliding down the loop toward the footpoints with an average speed of about 57 km s$^{-1}$, similar to the speeds of the descending cool clumps observed by Schrijver (2001).

Comparing our current analysis with the previous inversion technique employed by Andries et al. (2005b) and Van Doorsselaere et al. (2007) that neglected the effect of loop expansion, we now assume the magnetic field is constant ($\Gamma = 1$). This results in a density scale height of $H = 15.00$ Mm, which is an overestimation by a factor of 1.96. To quantitatively compare this with other possible effects, e.g., the line of sight error, we assume the loop is vertical ($\theta = 0°$). This actually gives a smaller overestimation factor of 1.47 and strongly illustrates the importance of measuring flux tube expansion.

Following Andries et al. (2005b), Van Doorsselaere et al. (2007) used *TRACE* 171 Å observations of kink oscillations to estimate the density scale heights of $H = 68$ Mm ($T = 1.45$ MK) and $H = 30$ Mm ($T = 0.64$ MK) for two neighboring loops in the postflare arcade of 2001 April 15, 21:58 UT (the so-called "harmonica event"). Unfortunately, the time series data for this event does not allow us to visually estimate the loop expansions, since at most, they are only visible at small apex sections (see Verwichte et al. 2004). Therefore, the scale height estimates of Van Doorsselaere et al. (2007) can only be taken as upper bounds. Interestingly, 171 Å emission measure analysis by Aschwanden & Terradas (2008) has shown that most of the oscillating loops in this postflare arcade are in a cooling phase. If we assume that $\Gamma$ is approximately the same for the neighboring loops studied by Van Doorsselaere et al. (2007) then the dissimilar diagnosed scale heights may be due to them being at different stages of cooling.

## 5. SUMMARY AND CONCLUSIONS

Previously, MHD wave theory of the standing fast kink mode allowed coronal loops to be modeled with arbitrary longitudinal plasma density. Recently, a practical theoretical advancement has been made, which as well as modeling a flux tube with arbitrary longitudinal plasma density, also includes the important effect of loop expansion. Theoretically, it has been shown that using the measurement of frequency ratio $\omega_2/\omega_1$ to estimate the coronal plasma density scale height without taking account of loop expansion causes the scale height to be overestimated. Implementing this recent MHD wave theory, in § 2 we have presented an improved magnetoseismological method to determine the coronal plasma density scale height, that corrects for the important effects of loop expansion, shape, and inclination. In the particular case study of § 3, it was demonstrated that if the effect of loop expansion were to be neglected it would cause a density scale height overestimation by a factor of approximately 2. It was also revealed in our case study that accurately estimating the loop's expansion was as important as determining other geometric properties such as loop inclination.

The method presented in this Letter offers an important refinement to previous magnetoseismological inversions to determine the longitudinal density structure of coronal loops. Our method also complements other novel magnetoseismological approaches that have been proposed to determine the transverse plasma structure of coronal loops by, e.g., Verwichte et al. (2006) and Arregui et al. (2007).

G. V. is grateful to the Engineering and Physical Sciences Research Council (EPSRC), UK, for funding received. R. E. acknowledges M. Keray for patient encouragement and is also grateful to NSF, Hungary (OTKA, Ref. No. K67746), for financial support. D. B. J. is funded by a Northern Ireland Department for Employment and Learning studentship. D. B. J. additionally thanks NASA Goddard Space Flight Center for a CAST studentship.

## REFERENCES

Andries, J., Arregui, I., & Goossens, M. 2005a, ApJ, 624, L57 (VE08)
Andries, J., Goossens, M., Hollweg, J. V., Arregui, I., & Van Doorsselaere, T. 2005b, A&A, 430, 1109
Arregui, I., Andries, J., Van Doorsselaere, T., Goossens, M., & Poedts, S. 2007, A&A, 463, 333
Aschwanden, M. J. 2004, Physics of the Solar Corona: An Introduction (Berlin: Springer)
Aschwanden, M. J., De Pontieu, B., Schrijver, C. J., & Title, A. M. 2002, Sol. Phys., 206, 99
Aschwanden, M. J., Fletcher, L., Schrijver, C. J., & Alexander, D. 1999, ApJ, 520, 880
Aschwanden, M. J., Nitta, N. V., Wuesler, J., & Lemen, J. R. 2008a, ApJ, 680, 1477
Aschwanden, M. J., & Terradas, J. 2008, ApJL, 686, L127 (http://solar.physics.montana.edu/cgi-bin/eprint/index.pl?entry=7969)
Aschwanden, M. J., Wüsler, J., Nitta, N. V., & Lemen, J. R. 2008b, ApJ, 679, 827
Aulanier, G., DeLuca, E. E., Antichos, S. K., McMullen, R. A., & Golub, L. 2000, ApJ, 540, 1126
De Moortel, I., & Brady, C. S. 2007, ApJ, 664, 1210
Díaz, A. J., Oliver, R., & Ballester, J. L. 2002, ApJ, 580, 550
Dymova, M. V., & Ruderman, M. S. 2005, Sol. Phys., 229, 79
———. 2006, A&A, 459, 241
Edwin, P. M., & Roberts, B. 1983, Sol. Phys., 88, 179
Erdélyi, R., & Verth, G. 2007, A&A, 462, 743
Jess, D. B., Mathioudakis, M., Erdélyi, R., Verth, G., McAteer, R. T. J., & Keenan, F. P. 2008, ApJ, 680, 1523
Klimchuk, J. A. 2000, Sol. Phys., 193, 53
Nakariakov, V. M., & Ofman, L. 2001, A&A, 372, L53
Nakariakov, V. M., Ofman, L., DeLuca, E. E., Roberts, B., & Davila, J. M. 1999, Science, 285, 862
Ofman, L. 2007, ApJ, 655, 1134
O'Shea, E., Srivastava, A. K., Doyle, J. G., & Banerjee, D. 2007, A&A, 473, L13
Roberts, B., Edwin, P. M., & Benz, A. O. 1984, ApJ, 279, 857
Ruderman, M. S. 2007, Sol. Phys., 246, 119
Ruderman, M. S., Verth, G., & Erdélyi, R. 2008, ApJ, 686, 694
Schrijver, C. J. 2001, Sol. Phys., 198, 325
Testa, P., Peres, G., & Reale, F. 2002, ApJ, 580, 1159
Uchida, Y. 1970, PASJ, 22, 341
Van Doorsselaere, T., Nakariakov, V. M., & Verwichte, E. 2007, A&A, 473, 959
Verth, G., & Erdélyi, R. 2008, A&A, 486, 1015
Verth, G., Van Doorsselaere, T., Erdélyi, R., & Goossens, M. 2007, A&A, 475, 341
Verwichte, E., Foullon, C., & Nakariakov, V. M. 2006, A&A, 452, 615
Verwichte, E., Nakariakov, V. M., Ofman, L., & Deluca, E. E. 2004, Sol. Phys., 223, 77
Watko, J. A., & Klimchuk, J. A. 2000, Sol. Phys., 193, 77